Symbol Error Outage Analysis of MIMO OSTBC Systems Over Rice Fading Channels in Shadowing Environments

Philippe Mary, Mischa Dohler, Jean-Marie Gorce and Guillaume Villemaud

Abstract

We deal with the analytical performance study of orthogonal space-time block codes (OSTBC) applied to multiple-input multiple-output (MIMO) systems experiencing non-frequency selective fast fading and shadowing. We address the problem of finding a tractable closed-form expression for the symbol error outage (SEO) over Rice fading channels in shadowing environments. In order to obtain the SEO, a symbol error probability (SEP) inversion w.r.t. the average signal to noise ratio (ASNR) is needed since no invertible SEP approximations are available in literature. We propose an accurate approximation for the SEP in Rice fading channels. This new approximation is proved to be invertible w.r.t. the ASNR and this expression thus facilitates the derivation of a tight approximation of the SEO in lognormal shadowing environments.

Index Terms

Symbol Error Probability, Symbol Error Outage, Rice Fading Channels, Shadowing, OSTBC, MIMO.

P. Mary, is with European University of Brittany, INSA, IETR, UMR 6164, bât. 6, 20 Avenue des Buttes de Coësmes, CS 70839, F-35708 Rennes Cedex 7, FRANCE (phone: +33-22-323-8592, emails: philippe.mary@insa-rennes.fr).

M. Dohler is with CTTC, Parc Mediterrani de la Tecnologia, Av. Carl Friedrich Gauss 7, 08860 Castelldefels, Barcelona, Spain (phone: +34-93-645-2900, fax: +34-93-645-2901, email: mischa.dohler@cttc.es).

J.M. Gorce and G. Villemaud are with CITI, INSA-Lyon, bat. Claude Chappe, 6 avenue des arts, 69621 Villeurbanne Cedex, France (emails: {jean-marie.gorce, Guillaume.Villemaud}@insa-lyon.fr).

I. Introduction

The demand on the quality of service (QoS) of wireless digital communication systems is continuously increasing. The paradigm of orthogonal space-time block codes multiple-input multiple-output (OSTBC MIMO) systems [1] to meet these demands is becoming reality [2]. The performance analysis of said systems within realistic propagation environments is key and fundamental for a proper network planning, optimization and deployment. Although spatial multiplexing techniques have been shown to exhibit better performances in term of outage capacities than transmit diversity transceivers [3], the OSTBC MIMO systems are still of interest for coverage-limited situations. Moreover, the OSTBC MIMO systems allow computational efficient decoders and hence energy-saving handset terminals.

Practical wireless systems, such as cellular networks, suffer from fast fading and lognormally distributed shadowing[1]. It is now well known that the symbol error probability (SEP), which is the common measure of QoS in fading channels [5], is not adapted to non-ergodic shadowing processes. In this case, outage is typically invoked leading to symbol error outage (SEO) probabilities [4], [6]. This criterion, defined as the probability that a certain average SEP cannot be supported for a given shadowing statistics, measures the QoS of wireless systems in shadowing environments. It is typically used by mobile operators when dimensioning their cellular networks.

The absence of mathematical closed form expressions for SEO has forced simulations to be used for network optimization problems, requiring often weeks of simulations [2]. In order to obtain the SEO, we need to inverse the SEP w.r.t. the average signal to noise ratio (ASNR). It is a difficult task and minimal research efforts have been granted to this subject. Conti *et al.* have proposed invertible tight lower and upper bounds of the SEP for M-PSK signals with maximum ratio combining (MRC) reception in [6]. Their bounds allow to find closed form expressions for the SEO in shadowing environments. This work extends a previous result obtained by the same authors in [4] on tight bounds for the bit error outage (BEO) of BPSK signals with diversity reception to M-QAM signals in [7], [8] for application to fast and slow adaptive modulation techniques with diversity in small-scale and large-scale fading. Recently, Conti *et al.* have discussed the general behavior of the error probability for systems employing M-QAM

[1]The fading over the wireless channel is assumed to change quickly compared to shadowing; therefore, this channel is referred to as fast fading [4].

constellation in fading channels with Gaussian disturbance [9]. They also discussed about local tight bounds for the SEP. In [10], [11], we have introduced a new approach to finding an invertible expression of the SEP based on the approximation of the hypergeometric function. We dealt with Nakagami-m fading channels and general coherent modulation (i.e. M-PSK, M-QAM) for single-input single-output (SISO) systems. Our results can also be extended to account for OSTBC MIMO systems by multiplying the Nakagami fading factor m with the number of transmit and receive antennas as well as performing some power scaling due to the use of OSTBCs of a given rate.

However, Nakagami-m fading does not model all occurring propagation environments. In particular, the Rician fading model matches with measurements in macro-cell or pico-cell presenting a line of sight (LOS) situation [12]. Moreover, although a Rician distribution can be approximated by a Nakagami-m distribution, they are fundamentally different. Indeed, the SEP versus ASNR in Rice fading channels is not a concave curve as it is the case for many fading channels [13], but exhibits an inflection point. Traditional asymptotic approaches are therefore not applicable and specific approximations dedicated to the Rician case need to be developed. Our contribution lies in the exposure of a novel approximation for the SEP in OSTBC MIMO systems and its inversion w.r.t. the ASNR, which is complicated by the fact that not even closed form solutions for the average SISO SEP exist. Our approximation holds for M-PSK and M-QAM signals and allows to evaluate the SEO in closed form with minimal computational effort.

This paper is organized as follows. In Section II, the MIMO system model is described. In Section III, we propose a tight approximation of the SEP for OSTBC systems in Rician fading channels, as well as an inverse SEP expression w.r.t. the ASNR. In Section IV, we give some numerical results on the SEO in shadowing environments and conclusions are drawn in Section V.

II. System Model

A. OSTBC MIMO Model

In an OSTBC MIMO system with n_t transmit and n_r receive antennas, the information symbols $\{d_i\}_{i=1}^{N}$, selected from the M-PSK or M-QAM constellation of order M, are encoded by a space-time block code defined by a $n_t \times p$ column orthogonal matrix $\mathcal{G}$ where the entries are linear combination of symbols and their conjugates. Since p symbol durations are necessary to transmit

N symbols, the rate R of the OSTBC is $R = N/p$. The transmission model is [1]:

$$\mathbf{Y} = \mathbf{H}\mathcal{G} + \mathbf{N}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is a flat fading MIMO channel where the entries $h_{k,l}$ are the independent and identically distributed (i.i.d.) complex channel gains between the $k-$th receive antenna and the $l-$th transmit antenna. In Rice fading channels, the MIMO channel matrix contains a deterministic term due to the specular component and a random component due to the scattering properties of the wireless channel. The matrix $\mathbf{Y} \in \mathbb{C}^{n_r \times p}$ is the signal received over the n_r reception antennas during p symbol times and $\mathbf{N} \in \mathbb{C}^{n_r \times p}$ is a noise matrix where elements are i.i.d. complex Gaussian random variables with a one-sided power spectral density N_0.

Under perfect channel state information assumption, the OSTBC scheme converts the matrix channel into a scalar one [1]. The output SNR of the maximum likelihood detector is $\gamma_{\text{OSTBC}} = ||\boldsymbol{H}||_F^2 \, \overline{\gamma}_s / n_t R$ and its moment generating function (MGF) can be expressed as [14], [15]:

$$M_{\gamma_{\text{OSTBC}}}(-s) = \frac{\exp\left(-\frac{sK\overline{\gamma}_s}{n_t R(1+K)+s\overline{\gamma}_s}\right)^{n_t n_r}}{\left(1 + s\frac{\overline{\gamma}_s}{n_t R(1+K)}\right)^{n_t n_r}}, \tag{2}$$

where K is the Rice factor linked with the specular component power, $\overline{\gamma}_s = E_m/N_0$ is the ASNR per symbol per receive antenna, averaged over small-scale fading, with E_m being the total energy transmitted and $||\boldsymbol{H}||_F^2$ is the square of the Frobenius norm of the matrix $\boldsymbol{H}$.

B. Symbol Error Outage in Log-Normal Shadowing Environments

In shadowing environment, the SEO, $P_s(O)$ (i.e. the probability that an outage event O is realized), is invoked and defined as the probability of an SEP exceeding a threshold, P_s^* [6]:

$$P_s(O) = Q\left(\frac{\mu_{dB} - 10\log_{10}\overline{\gamma}_s^{th}(P_s^*)}{\sigma_{dB}}\right), \tag{3}$$

where $Q(x)$ is the Gaussian Q-Function, $\overline{\gamma}_s^{th}(P_s^*)$ is the required ASNR to reach the target SEP P_s^* and parameters μ_{dB} and σ_{dB} are the mean and the standard deviation of ASNR considering shadowing, expressed in dB. This ASNR expression depending on the SEP can be obtained by: 1) numerically inverting the SEP expression via a look up table which is very tedious; 2) deriving an invertible approximation of the SEP in order to obtain a closed form of the ASNR as a function of the target SEP. The latter will be now pursued in subsequent sections.

III. Approximation and Inversion of the SEP for Rice fading channels

A. Exact SEP

For Rice fading channels, closed form solutions for average SEP do not exist, even for traditional coherent modulations (M-PSK and M-QAM) and SISO systems [5]. The exact form of the SEP is obtained by integrating the MGF of the output ASNR, $M_{\gamma_{\mathrm{OSTBC}}}$, given in (2). Hence, the exact average SEP over a flat Rice fading MIMO channel for M-PSK signals is:

$$P_s(E|\overline{\gamma}_s) = \frac{1}{\pi}\int_0^{(M-1)\pi/M} M_{\gamma_{\mathrm{OSTBC}}}\left(-\frac{g_{psk}}{\sin^2\theta}\right)d\theta, \tag{4}$$

and for M-QAM signals:

$$P_s(E|\overline{\gamma}_s) = \frac{4g}{\pi}\int_0^{\pi/2} M_{\gamma_{\mathrm{OSTBC}}}\left(-\frac{g_{qam}}{\sin^2\theta}\right)d\theta - \frac{4g^2}{\pi}\int_0^{\pi/4} M_{\gamma_{\mathrm{OSTBC}}}\left(-\frac{g_{qam}}{\sin^2\theta}\right)d\theta, \tag{5}$$

where $g_{psk} = \sin^2(\pi/M)$, $g_{qam} = 3/\left(2\left(M-1\right)\right)$, $g = 1 - 1/\sqrt{M}$ and M being the modulation order.

B. New Approximations of the SEP

The integrals above cannot be expressed by means of hypergeometric functions as previously done for Nakagami-m fading channels [11]. A different Laplace approach needs therefore to be used in order to find a tractable expression for the SEP. Proven in Appendix VI-A, the average SEP given in (4) and (5) can be expressed as per subsequent proposition.

Proposition 1 (SEP approximation) *In an i.i.d. MIMO Rice flat fading channel and with a ML OSTBC receiver, an invertible expression of the average SEP for M-PSK and M-QAM coherent modulations is:*

$$P_s\left(E|\overline{\gamma}_s\right) \approx \kappa\frac{\left((1+K)n_tR\right)^{n_tn_r}}{\sqrt{\pi n_tn_r}\left((1+K)n_tR + g_{mod}\overline{\gamma}_s\right)^{n_tn_r}}\exp\left(-\frac{g_{mod}Kn_tn_r\overline{\gamma}_s}{(1+K)n_tR + g_{mod}\overline{\gamma}_s}\right). \tag{6}$$

where g_{mod} is equal to g_{qam} or g_{psk} according to the considered modulation, and with κ being a constant equal to 1 or $4g$ for M-PSK or M-QAM signals respectively. This expression converges to the exact value for $n_tn_r \to \infty$ and $\overline{\gamma}_s \to \infty$.

The new result in Proposition 1 provides an invertible SEP approximation of M-ary signals in Rice fading channels. It can be verified from Figure 1 that the approximation in (6) is relatively tight at high SEP, i.e. $10^{-1} - 10^{-4}$, which is the region of interest without coding. The exact SEP and its approximation are plotted for 16 and 64-QAM signals, a 3×3 OSTBC MIMO system with a half-rate OSTBC and considering the Rayleigh fading case, $K = 0$ dB and $K = 10$ dB [2]. The error between the exact value and the approximation proposed is never larger than 0.5 dB for a SEP value between 10^{-1} and 10^{-5}.

C. Inversion of SEP

Proven in Appendix VI-B, above SEP expression can be inverted to yield the ASNR per receive antenna, $\overline{\gamma}_s$, in Rice fading channels as per subsequent proposition.

Proposition 2 (SEP inversion) *In an i.i.d. MIMO Rice flat fading channel, the closed form expression of the ASNR versus the average SEP is approximatively given by:*

$$\overline{\gamma}_s^{th}\left(P_s^*\right) \approx \frac{n_t R\left(1+K\right)}{g_{mod}} \left(\frac{K}{W_0\left(K e^K \left(\frac{\sqrt{\pi n_t n_r} P_s^*}{\kappa}\right)^{1/n_t n_r}\right)} - 1 \right), \quad (7)$$

where W_0 is the principal branch of the Lambert function which is defined to be the function satisfying $W(z)e^{W(z)} = z$.

This new result allows to estimate the SEO in shadowing environments as will be discussed in the following section. Although the expression of the ASNR w.r.t. to the target SEP involves a special function, i.e. the Lambert function, it can be evaluated in a fast manner [16].

IV. Numerical Results and Discussion

We analyze the accuracy of the proposed inversion by plotting the SEO of OSTBC systems in different fading channel and shadowing conditions. We generally plot two set of curves: exact and approximate. The first set is obtained by numerically inverting the exact SEP values w.r.t. the ASNR (i.e. (4) and (5)) and replacing the ASNR into (3). The second set is plotted by replacing the expression of the ASNR in (7) into (3). The target SEP used in the simulations is 10^{-2} unless otherwise mentioned. The SEO is plotted successively according to the shadowing mean, i.e. μ_{dB}, and the shadowing standard deviation, i.e. σ_{dB}, both in dB.

[2] For the Rayleigh fading case, a better approximation can be found in [10], [11] as a special case of the Nakagami-m distribution.

In Figures 2 and 3, we plot the SEO for two MIMO OSTBC system configurations, i.e. 2×2 and 3×3 respectively, and two modulation orders, i.e. 16 and 64-QAM. The specular Rice component takes two values, i.e. $K = 0$ dB and $K = 10$ dB. In Figure 2, the error between the herein proposed and the exact value of the SEO is never larger than 1 dB, whereas in Figure 3 the error is never larger than 0.4 dB. The worst precision is obtained for a 64-QAM signal and a 2×2 MIMO transceiver. Nevertheless, the larger the system size, the more accurate the approximation. This behavior comes from the Laplace approximation which is asymptotically tight (i.e. for $T \rightarrow \infty$ as per Appendix VI-A). In our case, when $n_t n_r$ increases, conditions for the Laplace approximation become more favorable and the approximation thus tighter. For example, for a 64-QAM signal, a 2×2 MIMO system, $K = 0$ dB and $\sigma_{dB} = 5$ dB, the approximated SEO is $5.5 \cdot 10^{-2}$ instead of $3.7 \cdot 10^{-2}$ for the exact value. For the same system settings but a 3×3 MIMO system, the approximated SEO is about $1.8 \cdot 10^{-3}$ and the actual value is $1.4 \cdot 10^{-3}$.

The proposed approximation of the SEP considering OSTBC systems and Rice fading channels is the only one known to date to the best of our knowledge and it is close to the exact value given the simplifications made. Moreover, the approximation is invertible and allows to estimate the SEO with the same accuracy. Although the evaluation of the SEO in Rice fading channels requires the computation of the principal branch of the special Lambert function W_0, it can be done with minimal computational effort. Since the Lambert W function is the solution of the equation $we^w = x$, it is natural to consider root finding methods to compute W, such as Newton or Halley's method. The latter is a third-order method for solving equations and has a faster convergence than Newton's method (first order) [17]. It takes the following form at the $(j+1)$-th iteration [16]:

$$w_{j+1} = w_j - \frac{w_j e^{w_j} - x}{e^{w_j}\left(w_j + 1\right) - \frac{(w_j+2)\left(w_j e^{w_j} - x\right)}{2w_j+2}}. \quad (8)$$

Each iteration requires 4 complex floating point multiplications (once e^{w_j} is evaluated), 2 floating point divisions and 6 additions. According to [17], the number of correct digits at the $j-$th step is roughly 3 times the number which were correct at the $(j-1)$-th step. Therefore, if d digits of precision are needed for the root computation, then only the last iteration needs be done on d digits, while the iteration before requires $d/3$ digits, the iteration before $d/9$ digits and so on. This dependency can be very useful in variable precision architectures in order to reduce complexity. Moreover, the evaluation of W_0 depends on some very few system parameters, i.e.

the MIMO system size, the modulation scheme (κ), the fading channel K and the target SEP P_s^*. It is clearly less complex to evaluate W_0 than producing a look-up table of the SEP according to the ASNR for all possible system configurations.

These results can therefore be used by mobile operators when dimensioning their cellular networks, where traditional approaches require lengthy simulations [2]. The second application is the link adaptation under a QoS criterium as proposed by Conti *et al.* in [8]. In the open literature, link adaptation is often based on an average SNR at the receiver. Our results allow to set a transmit power according to a target SEP under a certain outage, which is directly related to the user's QoS.

Coded systems are not considered in this paper. However, we investigated SEP of interest between 10^{-1} and 10^{-4} which is the rough SEP of practical situations before the decoding process. A channel coding scheme implies generally an SNR-shift to the left of the SEP curves. Hence, our result can be used as a rough estimation of the outage probability before channel decoding. For capacity-limited systems, the target bit error probability can be linked to the ϵ-channel capacity [18], [19], i.e. the spectral efficiency achievable under the target error probability ϵ. The study of the outage ϵ-capacity under shadowing conditions is largely unexplored and would deserve some attentions from the scientific community.

V. Concluding Remarks

We have studied the performance of OSTBC MIMO systems over flat Rice fading channels and in shadowing environments. We derived a new approximation of the average SEP for M-PSK as well as M-QAM signals. The approximation is accurate over a large range of ASNRs and fading severity values as well as several OSTBC MIMO system configurations. The proposed expression is invertible w.r.t. the ASNR. From the new SEP approximation, we proposed a mathematical closed form expression of the ASNR per receive antenna $\overline{\gamma}_s$ as a function of the target SEP. The latter allowed obtaining the SEO in a closed form for a given log-normally distributed shadowing environment. The problem of finding a tractable expression for the SEO taking into account composite channels, i.e. with fading and shadowing (slow fading channels), is largely unsolved and hence left for future work. Moreover, the analysis of error outage probabilities for coded systems in shadowing environments is a challenging task and a really important matter for wireless network operators and will be addressed in a future work.

VI. Appendix

A. Proof of Proposition 1

Integrals as $I = \int_a^b h(\theta) e^{-Tg(\theta)} d\theta$ can be approached thanks to the Laplace method as $\tilde{I} = \sqrt{\frac{2\pi}{Tg''(\theta_0)}} h(\theta_0) e^{-Tg(\theta_0)}$ [20], where g'' is the second derivative of g over $[a,b]$, g has an unique minimum in $\theta_0 \in [a,b]$, $h(\theta_0) \neq 0$ and $h(\theta)$ has a constant sign over $[a,b]$. Moreover, $\tilde{I}$ is converging to I, as $T \to \infty$. For the integral in (4), let us take the following base of functions:

$$h(\theta) = \frac{1}{\pi} \qquad \text{and} \qquad g(\theta) = -\ln\left(\frac{\exp\left(-\frac{g_{psk} K \overline{\gamma}_s}{n_t R (1+K)\sin^2\theta + g_{psk}\overline{\gamma}_s}\right)}{\left(1 + \frac{g_{psk}\overline{\gamma}_s}{n_t R(1+K)\sin^2\theta}\right)}\right), \tag{9}$$

where $T = n_t n_r$. The point $\theta_0 \in [0, (M-1)\pi/M]$ where g is minimal is obtained by solving the relation $g'(\theta) = 0$. After some calculus, we get $\theta_0 = \pi/2$. Moreover, we have:

$$g''(\theta)\big|_{\theta=\frac{\pi}{2}} = \frac{2 g_{psk}\overline{\gamma}_s \left(n_t R (1+K)^2 + g_{psk}\overline{\gamma}_s\right)}{\left(n_t R(1+K) + g_{psk}\overline{\gamma}_s\right)^2}, \tag{10}$$

$$g(\theta)\big|_{\theta=\frac{\pi}{2}} = -\ln\left(\frac{(1+K)n_t R}{(1+K)n_t R + g_{psk}\overline{\gamma}_s} \exp\left(-\frac{g_{psk} K \overline{\gamma}_s}{(1+K)n_t R + g_{psk}\overline{\gamma}_s}\right)\right). \tag{11}$$

Replacing (10) and (11) into $\tilde{I}$, we obtain the Laplace approximation of the SEP for M-PSK:

$$P_s^{La}(E|\overline{\gamma}_s) \approx \frac{((1+K)n_t R)^{n_t n_r}}{\sqrt{\pi n_t n_r g_{psk}\overline{\gamma}_s \left(g_{psk}\overline{\gamma}_s + n_t R (1+K)^2\right)}} \frac{\exp\left(-\frac{g_{psk} K n_t n_r \overline{\gamma}_s}{(1+K)n_t R + g_{psk}\overline{\gamma}_s}\right)}{\left((1+K)n_t R + g_{psk}\overline{\gamma}_s\right)^{n_t n_r - 1}}. \tag{12}$$

In order to make (12) invertible w.r.t. the ASNR, the SEP needs to be expressed as $we^w = f(w)$, $w \in \mathbb{R}$, where the reciprocal function of f is the Lambert function W. Hence, we approximate the denominator of (12) appropriately by $\sqrt{\pi n_t n_r}\left((1+K)n_t R + g_{psk}\overline{\gamma}_s\right)^{n_t n_r}$. Hence, (12) is directly transformed into (6) which completes the proof for M-PSK signals. There is a supplementary gap between (12) and (6). By denoting P_s^{inv} the invertible SEP approximation in (6), the relative error between these two expressions, i.e. (12) and (6), is:

$$\left|\frac{P_s^{La}(E|\overline{\gamma}_s) - P_s^{inv}(E|\overline{\gamma}_s)}{P_s^{La}(E|\overline{\gamma}_s)}\right| = \left|1 - \frac{\sqrt{g_{mod}\overline{\gamma}_s\left(g_{mod}\overline{\gamma}_s + n_t R (1+K)^2\right)}}{(1+K)\, n_t R + g_{mod}\overline{\gamma}_s}\right|. \tag{13}$$

Under asymptotic conditions, i.e. $\overline{\gamma}_s \to \infty$, the error tends to zero.

For M-QAM constellations, two integrals are involved in (5). The expression in (5) can be lower bounded very tightly by:

$$P_s(E|\overline{\gamma}_s) \geq \frac{4g}{\pi} \int_{\pi/4}^{\pi/2} M_{\gamma_{\mathrm{OSTBC}}}\left(-\frac{g_{qam}}{\sin^2\theta}\right) d\theta. \tag{14}$$

As M is increasing, g^2 is converging to g and the lower bound in (14) is become tighter, but remains very close to the exact value for small M. The Laplace method is hence applied to the integral in (14) and a similar approximation as the one performed for M-PSK signals is introduced, which completes the proof for M-QAM signals.

B. Proof of Proposition 2

Multiplying (6) by $\sqrt{\pi n_t n_r}/\kappa$ and taking the $n_t n_r-$th root we have:

$$\left(\frac{\sqrt{\pi n_t n_r}P_s^*}{\kappa}\right)^{\frac{1}{n_t n_r}} \approx \frac{(1+K)n_t R}{(1+K)n_t R + g_{mod}\overline{\gamma}_s} \exp\left(-\frac{g_{mod}K\overline{\gamma}_s}{(1+K)n_t R + g_{mod}\overline{\gamma}_s}\right). \tag{15}$$

The term in the exponential can be rewritten as:

$$\exp\left(-\frac{g_{mod}K\overline{\gamma}_s}{(1+K)n_t R + g_{mod}\overline{\gamma}_s}\right) = \exp\left(\frac{K(1+K)n_t R}{(1+K)n_t R + g_{mod}\overline{\gamma}_s}\right)\exp\left(-K\right). \tag{16}$$

Replacing the above expression into (15) and multiplying each side of the equation by Ke^K:

$$Ke^K\left(\frac{\sqrt{\pi n_t n_r}P_s^*}{\kappa}\right)^{\frac{1}{n_t n_r}} \approx \frac{K(1+K)n_t R}{(1+K)n_t R + g_{mod}\overline{\gamma}_s} \exp\left(\frac{K(1+K)n_t R}{(1+K)n_t R + g_{mod}\overline{\gamma}_s}\right). \tag{17}$$

Let's define $Z = K\left(1+K\right)n_t R/\left(\left(1+K\right)n_t R + g_{mod}\overline{\gamma}_s\right)$. The solution of $f\left(P_s^*\right) = Ze^Z$ is given by the Lambert function as:

$$\frac{K(1+K)n_t R}{(1+K)n_t R + g_{mod}\overline{\gamma}_s} = W_0\left(Ke^K\left(\frac{\sqrt{\pi n_t n_r}P_s^*}{\kappa}\right)^{\frac{1}{n_t n_r}}\right). \tag{18}$$

From (18) and with some calculus, $\overline{\gamma}_s = f\left(P_s^*\right)$ is obtained and the proof is complete.

REFERENCES

[1] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-Time Block Codes From Orthogonal Designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.

[2] M. J. Nawrocki, M. Dohler, and A. H. Aghvami, *Understanding UMTS Radio Network. Modelling, Planning and Automated Optimisation*. Chippenham, England: John Wiley & sons, Ltd, 2006.

[3] A. Lozano and N. Jindal, "Transmit Diversity vs. Spatial Multiplexing in Modern MIMO Systems," *IEEE Transactions on Wireless Communications*, vol. 9, no. 1, pp. 186 –197, jan. 2010.

[4] A. Conti, M. Z. Win, M. Chiani, and J. H. Winters, "Bit Error Outage for Diversity Reception in Shadowing Environment," *IEEE Communications Letters*, vol. 7, no. 1, pp. 15–17, Jan. 2003.

[5] M. K. Simon and M. S. Alouini, *Digital Communication Over Fading Channels*, 2nd ed. New Jersey, USA: Wiley-IEEE Press, 2005.

[6] A. Conti, M. Z. Win, and M. Chiani, "On the Inverse Symbol-Error Probability for Diversity Reception," *IEEE Transactions on Communications*, vol. 51, no. 5, pp. 753–756, May 2003.

[7] ——, “Invertible Bounds for M-QAM in Rayleigh Fading,” *IEEE Transactions on Wireless Communications*, vol. 4, no. 5, pp. 1994–2000, Sep. 2005.

[8] ——, “Slow Adaptive M-QAM With Diversity in Fast Fading and Shadowing,” *IEEE Transactions on Communications*, vol. 55, no. 5, pp. 895–905, May 2007.

[9] A. Conti, D. Panchenko, S. Sidenko, and V. Tralli, “Log-Concavity Property of the Error Probability With Application to Local Bounds for Wireless Communications,” *IEEE Transactions on Information Theory*, vol. 55, no. 6, pp. 2766 –2775, june 2009.

[10] P. Mary, M. Dohler, J. M. Gorce, G. Villemaud, and M. Arndt, “BPSK Bit Error Outage over Nakagami-m Fading Channels in Lognormal Shadowing Environments,” *IEEE Communications Letters*, vol. 11, no. 7, pp. 565–567, Jul. 2007.

[11] ——, “M-ary Symbol Error Outage Over Nakagami-m Fading Channels in Shadowing Environments,” *IEEE Transactions on Communications*, vol. 57, no. 10, pp. 2876–2879, Oct. 2009.

[12] R. J. C. Bultitude, S. A. Mahmoud, and W. A. Sullivan, “A Comparison of Indoor Radio Propagation Characteristics at 910 MHz and 1.75 GHz,” *IEEE Journal on Selected Areas in Communications*, vol. 7, no. 1, pp. 20–30, Jan. 1989.

[13] S. Loyka, V. Kostina, and F. Gagnon, “Symbol Error Rates of Maximum-Likelihood Detector: Convex/Concave Behavior and Applications,” in *Information Theory, 2007. ISIT 2007. IEEE International Symposium on*, June 2007, pp. 2501 –2505.

[14] H. Shin and J. H. Lee, “Performance Analysis of Space-Time Block Codes Over Keyhole Nakagami-m Fading Channels,” *IEEE Transactions on Vehicular Technology*, vol. 53, no. 2, pp. 351–362, Mar. 2004.

[15] Y. Ma and L. Zhao, “Achievable Performance of Orthogonal STBC Over Spatially Correlated Rician Channels,” *IEEE Transactions on Vehicular Technology*, vol. 56, no. 3, pp. 1251–1261, May 2007.

[16] R. Corless, G. Gonnet, D. Hare, and D. Jeffrey, “On the Lambert W function,” *Advances in Computational Mathematics*, vol. 5, pp. 329–359, 1996.

[17] G. Alefeld, “On the convergence of the Halley’s method,” *American Mathematical Monthly*, vol. 88, no. 7, pp. 530–536, 1981.

[18] G. Caire and K. Kumar, “Information theoretic foundations of adaptive coded modulation,” *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2274 –2298, dec. 2007.

[19] S. Verdu and T. S. Han, “A general formula for channel capacity,” *IEEE Transactions on Information Theory*, vol. 40, no. 4, pp. 1147 –1157, jul. 1994.

[20] R. W. Butler and A. T. A. Wood, “Laplace Approximations for Hypergeometric Functions with Matrix Argument,” *Annals of Statistics*, vol. 30, no. 4, pp. 1155–1177, 2002.

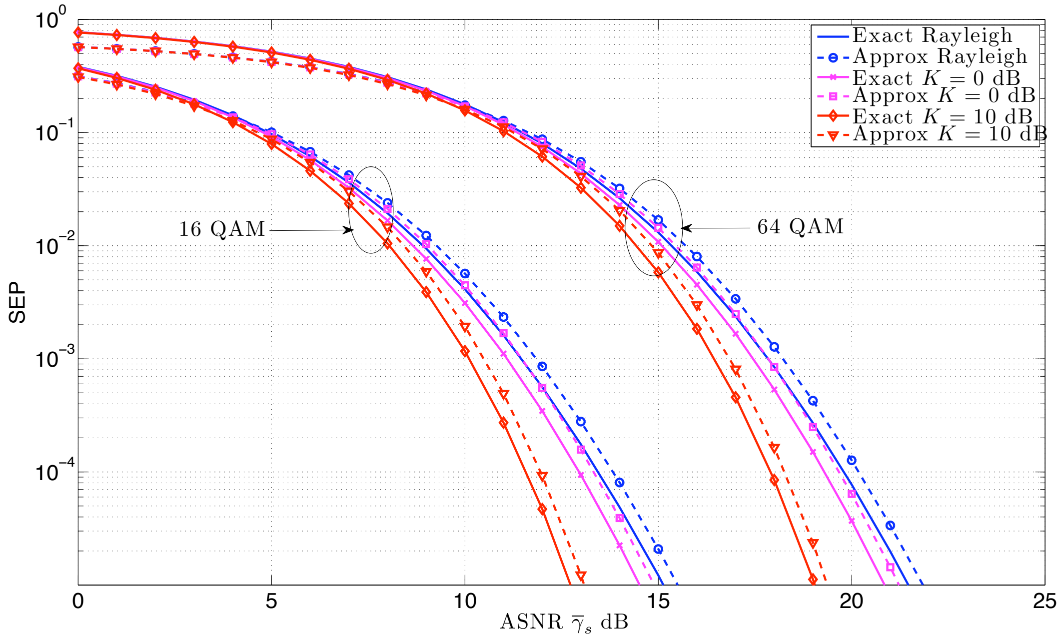
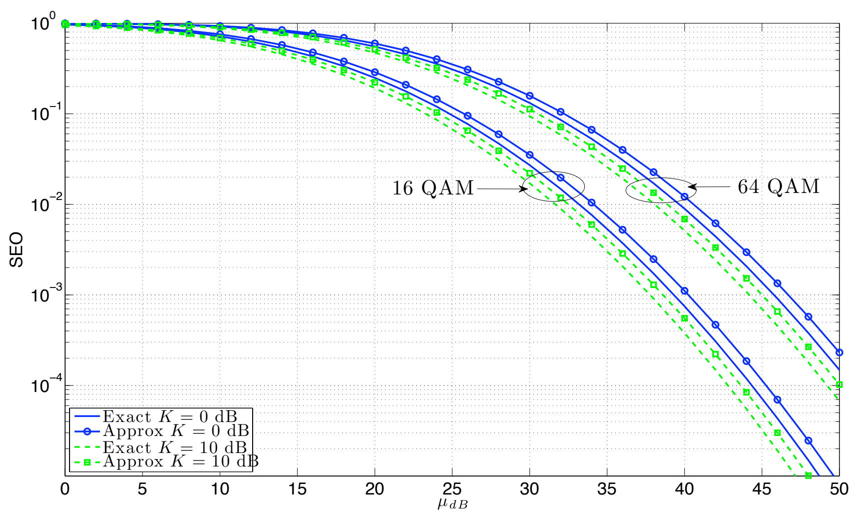
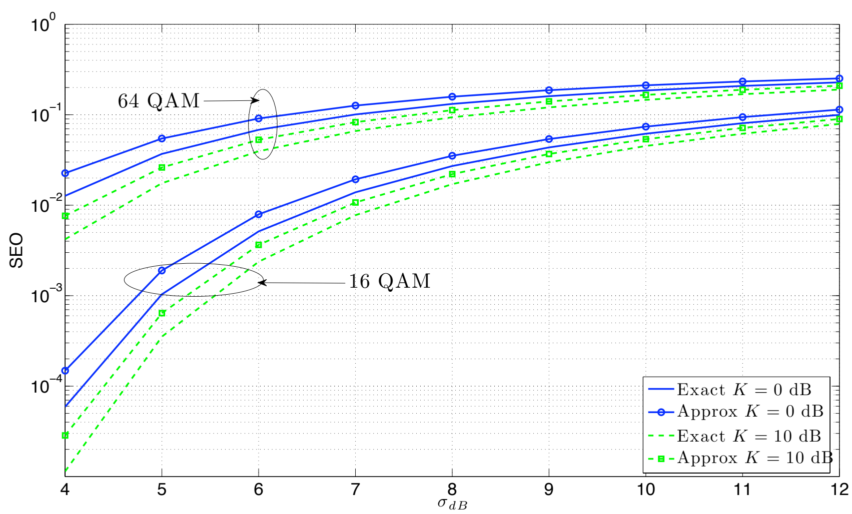


Figure 1. SEP of 16 and 64-QAM signals for a 3×3 OSTBC MIMO system in a Rice fading channel.

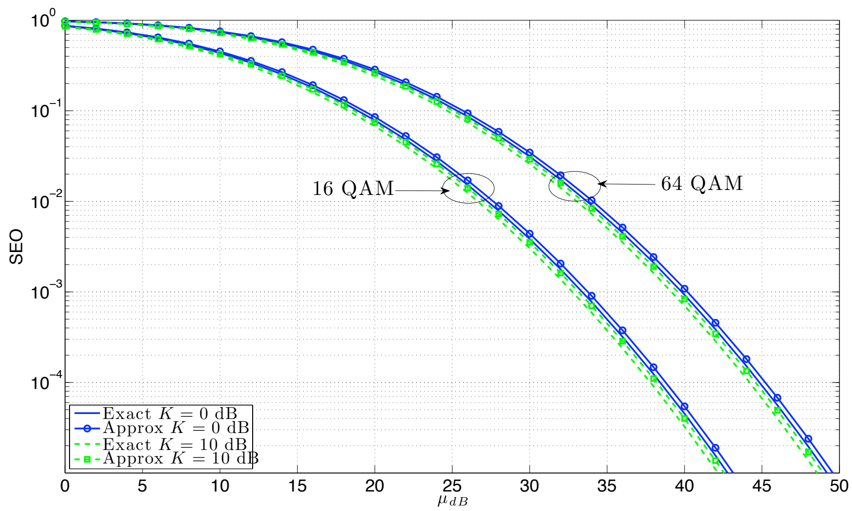


(a) SEO = $f(\mu_{dB})$; $\sigma_{dB} = 8$ dB

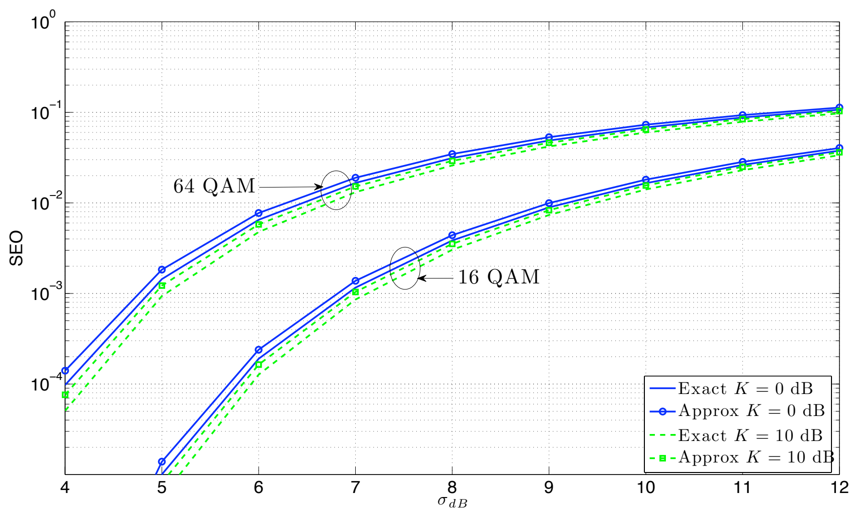


(b) SEO = $f(\sigma_{dB})$; $\mu_{dB} = 30$ dB

Figure 2. SEO versus mean 2(a) and standard deviation 2(b) of shadowing for a 2×2 MIMO OSTBC system and two modulation orders (i.e. 16-QAM, 64-QAM), labeled on Rice fading severity K.



(a) SEO = $f(\mu_{dB})$; $\sigma_{dB} = 8$ dB



(b) SEO = $f(\sigma_{dB})$; $\mu_{dB} = 30$ dB

Figure 3. SEO versus mean 3(a) and standard deviation 3(b) of shadowing for a 3×3 MIMO OSTBC system and two modulation orders (i.e. 16-QAM, 64-QAM), labeled on Rice fading severity K.