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ACE Course on MIMO Communication Systems and Antennas, KTH, Stockholm, Sept 5-9, 2005.

REFERENCES

The sources of this seminar are mainly:

- T.M. Cover and J.A. Thomas, Elements of Information Theory, Wiley Series in Telecommunications, John Wiley & Sons, New York, 1991.
- D. Tse, P.Wiswanath, *Fundamentals of Wireless Communications*, Cambridge University Press, 2005. Chaps 5,6,10.
- A. Goldmsith et al, Capacity Limits of MIMO Channels, IEEE Trans. on Selected Areas in Communications, Vol. 21, No. 5, June 2003.
- H. Boche and M. Wiczanowski, Queueing Theoretic Optimal Scheduling for Multiple Input Multiple Output Multiple Access Channel, ISSPIT 2003, Darmstadt, Germany.

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MOTIVATION

- Best way for multiple users to transmit over a shared medium? Orthogonal access? Simultaneous?
- Differences between uplink (multiple access) and downlink (broadcast) channels?
- Impact of multiple transmit and/or multiple receive antennas?
- In multi-user systems, can we take advantage of fading?
- Can the scheduling process be enhanced with channel-related information?
- Combined use of queue and channel information for scheduling?
- □ Information theory approach: keep it general !!

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OUTLINE

- Motivation
- A review of capacity issues in single-user systems
 - ✓ Definition, Capacity for MIMO systems.
- Capacity issues in multi-user systems:
 - ✓ Broadcast (BC) and Multiple Access (MAC) channels.
 - ✓ Capacity regions for SISO BC & MAC. Sum capacity. Symmetric capacity.
 - \checkmark Multi-user diversity. Channel-aware scheduling.
 - √ Fairness issues: Proportional Fair Scheduling
 - ✓ Slow-fading channels: Opportunistic Beamforming
- - Motivation
- □ Q&A

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- Motivation
- □ A review of capacity issues in single-user systems
 - ✓ Definition, Capacity for MIMO systems.

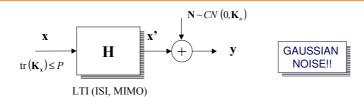
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A REVIEW OF CAPACITY ISSUES IN SINGLE-USER SYSTEMS

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CAPACITY IN LINEAR TIME INVARIANT SYSTEMS



•Definition of mutual information

$$I(X;Y) = h(X) - h(X|Y)$$
 with $h(X) = E_x(-\log f_x(x))$
 $h(X|Y) = E_{xy}(-\log f_{x|y}(x|y))$

• Information capacity of an AWGN channel with power constraint P:

$$C = \max_{f_x(x)} I(X; Y)$$
s.t. $\operatorname{tr}(\mathbf{K}_x) \le P$

• Mutual information maximized for GAUSSIAN input:

$$\mathbf{X} \sim CN\left(0, \mathbf{K}_{x}\right)$$

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CAPACITY IN LINEAR TIME INVARIANT SYSTEMS

• In these conditions, maximizing mutual information amounts to:

$$C = \max_{f_x(x)} I(X;Y) = \max_{\mathbf{K}_x} I(X;Y) = \max_{\mathbf{K}_x} \log_{\mathbf{K}_x} \mathbf{H}^H$$

- Remarks:
 - In general, \mathbf{K}_{x} depends on \mathbf{H} and what information is available @ Tx side (partial, full, none).
 - Units: bits/s/Hz...when log = log₂
 - Interpretation (Shannon's Channel Capacity Theorem): For every data rate R...

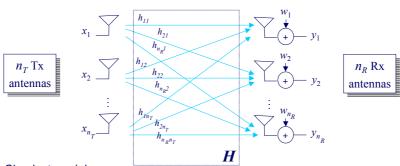
Information capacity (C) provides an <u>upper bound</u> of the achievable data rates (R)

- Assumptions: Gaussian input symbols & ideal channel coding (and decoding)
- Useful equivalence:

$$C = \max_{\mathbf{K}_{x}} \log \frac{\left|\mathbf{K}_{n} + \mathbf{H}\mathbf{K}_{x}\mathbf{H}^{H}\right|}{\left|\mathbf{K}_{n}\right|} = \max_{\mathbf{K}_{x}} \log \left|\mathbf{I} + \mathbf{K}_{n}^{-1}\mathbf{H}\mathbf{K}_{x}\mathbf{H}^{H}\right|$$

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MIMO CHANNEL MODEL



- · Simplest model:
 - Channel: Flat fading (frequency), static / independent Rayleigh fading (time)
 - $\bullet\quad \text{Noise: Gaussian (spatially) white}\quad \mathbf{N}\sim CN\,\left(0,\mathbf{K}_{\scriptscriptstyle n}\right) \rightarrow \mathbf{W}\sim CN\,\left(0,N_{\scriptscriptstyle o}I_{\scriptscriptstyle n_{\scriptscriptstyle g}}\right)$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_R} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n_T} \\ h_{21} & h_{22} & \cdots & h_{1n_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_R1} & h_{n_R2} & \cdots & h_{n_Rn_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_T} \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n_R} \end{bmatrix}$$
 $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$

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CAPACITY OF MIMO SYSTEMS (LTI)

$$C = \max_{\mathbf{K}_{x}} \log \left| \mathbf{I} + \mathbf{K}_{w}^{-1} \mathbf{H} \mathbf{K}_{x} \mathbf{H}^{H} \right| \qquad \mathbf{K}_{w} = N_{o} \mathbf{I}_{n_{R}}$$

• SISO, Shannon Capacity

$$\mathbf{K}_{x} = P$$

$$(\mathbf{K}_{w} = N_{o})$$

$$C = \log\left(1 + \frac{P|h|^{2}}{N_{o}}\right) = \log(1 + \text{SNR})$$

• MIMO, no CSI at Tx – Isotropic transmission:

$$\mathbf{K}_{x} = \frac{P}{n_{T}} \mathbf{I}_{n_{T}}$$

$$C = \log \left| \mathbf{I} + \frac{P}{N_{o} n_{T}} \mathbf{H} \mathbf{H}^{H} \right|$$

$$C \approx n \log \frac{P}{N_{o} n_{T}} + \sum_{i=1}^{n} \log \lambda_{i}^{2}$$

• MIMO, full CSI at Tx - Waterfilling over channel eigenmodes (SVD):

$$\mathbf{K}_{x} = \mathbf{V} \operatorname{diag}(P_{1} \dots P_{n}) \mathbf{V}^{H}$$

$$\mathbf{H} = \mathbf{U} \operatorname{diag}(\lambda_{1} \dots \lambda_{n}) \mathbf{V}^{H}$$

$$C = \sum_{i=1}^{n} \log \left[1 + N_{0}^{-1} \lambda_{i}^{2} P_{i} \right]$$

Power allocation (Lagrange): $P_i(\lambda_i) = \left(\mu - \frac{N_0}{\lambda_i^2}\right)^+ i = 1...n \sum_{i=1}^n P_i = P \quad n = \min(n_T, n_R)$

Asympt growth

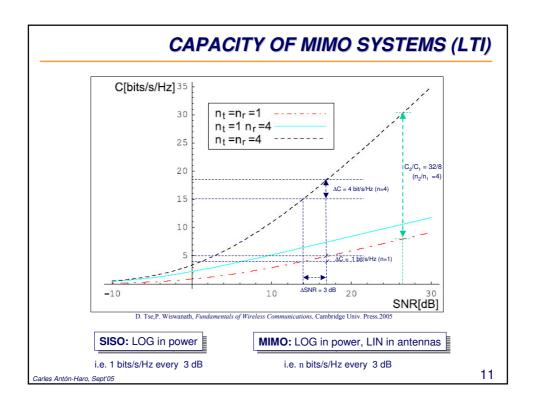
LOG in power

i.e. 1 bits/s/Hz every 3 dB

Asympt growth

LOG in power LIN in antennas

i.e. n bits/s/Hz every 3 dB



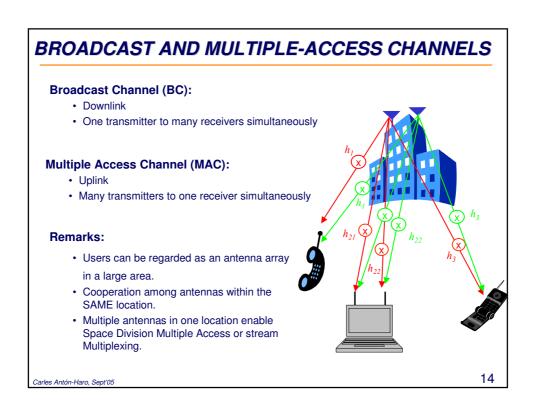
OUTLINE

- Motivation
- A review of capacity issues in single-user systems
 - Definition, Capacity for MIMO systems, time-varying systems.
- Capacity issues in multi-user systems:
 - ✓ Broadcast (BC) and Multiple Access (MAC) channels.
 - Capacity regions for SISO BC & MAC. Sum capacity. Symmetric capacity.
 - Multi-user diversity. Channel-aware scheduling.
 - ✓ Fairness issues: Proportional Fair Scheduling
 - \checkmark Slow-fading channels: Opportunistic Beamforming
 - Capacity regions for MIMO BC & MAC. Duality principle.

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CAPACITY ISSUES IN MULTI-USER SYSTEMS

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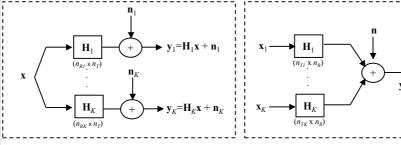


MIMO BC and MAC - CHANNEL MODEL

- One base station (BS) equipped with $n_T(n_R)$ antennas
- K user equipments (UE) equipped with $n_{Rk} (n_{Tk})$ antennas each

BS Shared power constraint

UE individual power constraints



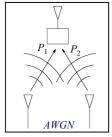
Broadcast Channel (BC)

Multiple Access Channel (MAC)

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CAPACITY REGION FOR MAC-AWGN



SISO, MAC, AWGN channel, K=2 users:

$$y[m] = x_1[m] + x_2[m] + w[m]$$

- Single user: Rate R achievable iff R<C → C upper perf. bound
- **Multi-user:** UEs communicate with BS in a shared bandwidth → trade-offs turning up!!
 - Set of achievable rates (R_1,R_2) with simultaneous communication??

 $R_1 + R_2 \le \log\left(1 + \frac{P_1 + P_2}{N_0}\right)$ $R_2 \le \log\left(1 + \frac{P_2}{N_0}\right)$

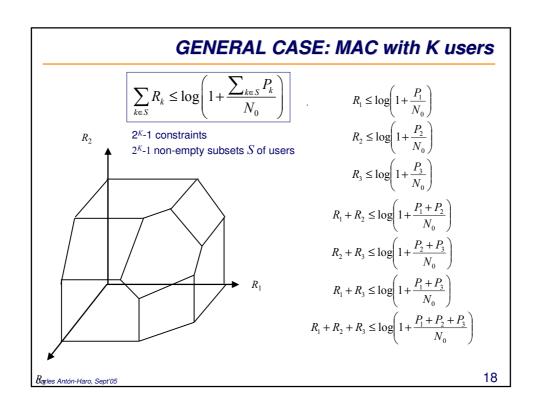
CAPACITY REGION, $\mathcal{C}!!$



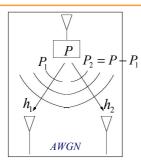
- Characterizes *optimal* trade-off achievable by *any* MA scheme.
- User 2 gets R₂>0 while user 1 attains singleuser bound (A) !!
- HOW? Successive interference Cancellation (SIC).
- Reversing detection order leads to different rate split (B) - fairness

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MEASURES OF INTEREST • Some performance measures (scalars) for a capacity region: Sum capacity $C_{\text{sum}} := \max_{(R_1, R_2) \in \mathcal{C}} R_1 + R_2$ Reached at AB segment (ANY point) Points A,B achievable via SIC M Intermediate points in AB via time/freq sharing Operating point TBD according to priorities or fairness constraints OPTIMAL OPERATING POINTS FOR SUM CAPACITY $R_1 + R_2 = \max = \log \left(1 + \frac{P_1 + P_2}{N_0} \right)$ Best policy in MAC-SISO: ALL users at a time (+ SIC) · Symmetric capacity Reached @ boundary (near/far) - C



CAPACITY REGION FOR BC-AWGN



SISO, BC, AWGN channel, K=2 users:

$$y_1[m] = h_1 x[m] + w_1[m]$$
 $y_2[m] = h_2 x[m] + w_2[m]$

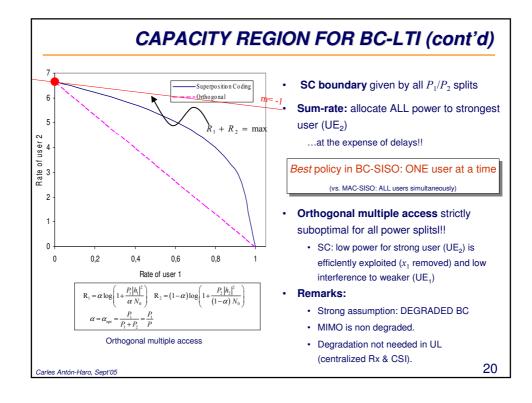
- BS communicates with UE in a shared bandwidth & shared $\underline{power}(P) \rightarrow trade-offs turning up!!$
 - How to MUX data for both users at the BS? x[m] = ??
 - Set of achievable rates (R₁,R₂) with simultaneous comms.??
- Assume: User 2 is the "strongest" $(|h_2| \ge |h_1|)$ and superposition coding $x[m] = x_1[m] + x_2[m]$
- If x_1 decodable at UE₁ (weakest) in the presence of x_2 , so is at UE₂ (strongest) for all power splits $P_{1,}P_{2}$ (not possible if reversed order)

$$SNIR_{x_1 \oplus UE_1} = \frac{P_1 |h_1|^2}{(1 - P_1)|h_1|^2 + N_0} \le \frac{P_1 |h_2|^2}{(1 - P_1)|h_2|^2 + N_0} = SNIR_{x_1 \oplus UE_2}$$

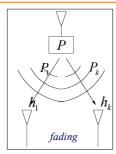
So apply SIC at the strongest (UE₂) and

$$R_{1} = \log \left(1 + \frac{P_{1}|h_{1}|^{2}}{(P - P_{1})|h_{1}|^{2} + N_{0}} \right) \qquad R_{2} = \log \left(1 + \frac{(P - P_{1})|h_{2}|^{2}}{N_{0}} \right)$$

$$R_{2} = \log \left(1 + \frac{(P - P_{1})|h_{2}|^{2}}{N_{0}} \right)$$



BC CHANNEL WITH FADING



SISO, BC, fading channel, K users:

$$y_k[m] = h_k[m]x[m] + w_k[m]$$

- Assumptions:
 - Fading processes ($\{h_k[m]\}$): Independent and identically distributed (symmetric case).
 - Power constraint (pooled power) : $E_H \left[\sum_{k=1}^{K} P_k[m] \right] = P$
- Take the case with CSIT (i.e power allocation possible):
 - AWGN: Sum capacity maximized by transmitting to the BEST user
 - Fading: Schedule the BEST user at EACH time (greedy approach). Equivalent point-to-point channel

$$\left|h\right|_{\mathrm{eq}}^{2} = \max_{k=1..K} \left|h_{k}\right|^{2}$$

How to allocate power? Temporal waterfilling for the equivalent P2P channel

$$P^*(\mathbf{h}) = \left(\frac{1}{\lambda} - \frac{N_0}{\max_{k=1...K} \left| h_k \right|^2} \right)^{+}$$



$$P^*(\mathbf{h}) = \left(\frac{1}{\lambda} - \frac{N_0}{\max_{k=1..K} \left| h_k \right|^2} \right)^+ \qquad C_{\text{sum}} = \mathbf{E}_h \left[\log \left(1 + \frac{P^*(\mathbf{h}) \left(\max_{k=1..K} \left| h_k \right|^2 \right)}{N_o} \right) \right]$$

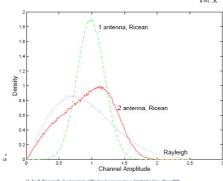
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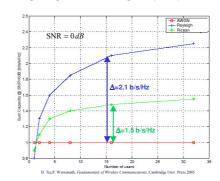
MULTI-USER DIVERSITY (MUDIV) GAIN

With K users FADING INDEPENDENTLY and OPPORTUNISTIC (DYNAMIC) SCHEDULING, channel gain improves

$$\left|h_{1}\right|^{2} \rightarrow \left|h\right|_{\text{eq}}^{2} = \max_{k=1..K} \left|h_{k}\right|^{2}$$

Higher gain means higher (sum) rate!!





- Gain wrt AWGN for K>1 (mid-high SNR)
- The amount of MUDiv increases with pdfs' tails: Rayleigh > Rice (κ =5, LOS, less "random")
- MUDiv gain increases with nr. of users (K): the stronger is the strongest channel

MULTI-USER vs. CLASSICAL DIVERSITY

· Purpose:

- Classical (time/frequency/space): Increase link *reliability* (slow fading)
- MUDiv: Increase average cell throughput (fast fading)
 - ...but no rate guarantees in specific fading states

Means:

- Classical: Counteract adverse fading effects.
- MUDiv: *Exploit* independent fading (capture strongest user)

Scope:

- Classical: Works at the link level
- MUDiv: System-wide (active users)

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REMARKS ON MUDIV

Signalling:

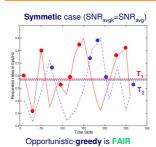
- UEs: Track their link quality (common pilot)
- BS: Access to quality measurements (feedback channel)
- Delay in the feedback channel (ass.: delay&error free)
 - · Mismatch actual channel-measured channel
 - FIX: \downarrow scheduling slots \Rightarrow \uparrow signalling overhead \Rightarrow selective MUDiv (f/b iff above threshold)

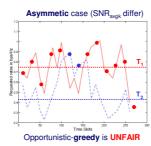
• Fairness & delay:

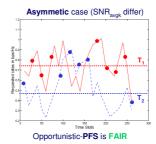
- Non-homogeneous user set in real-world networks (assumed so far)
 - $\bullet \ \, {\rm Different\ statistics\ (Rayleigh,\ Rice, ...)\ average\ SNRs\ (near-far)..\ RESOURCE\ ALLOCATION\ ??}$
- FIX: Proportional Fair Scheduler (PFS)

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PROPORTIONAL FAIR SCHEDULING (PFS)







Proportional Fair Scheduler: Schedule user with peak rate with respect to its average rate

$$k^*[m] = \max_{k} \frac{R_k[m]}{T_k[m]}$$

$$T_{k}[m] = \begin{cases} (1 - 1/t_{c})T_{k}[m] + (1/t_{c})R_{k}[m] & k = k^{*}, \\ (1 - 1/t_{c})T_{k}[m] & k \neq k^{*} \end{cases}$$

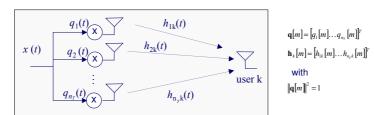
- PFS vs. greedy opportunistic schedulers:
 - Both channel-dependent (vs.round-robin, vs. queue-based). PFS implemented in IS-856.
 - Greedy: No short-term fairness, captures MUDiv, maximizes average sum-rate.
 - PFS: No short-term fairness, long-term fairness (same # access), captures some MUDiv, loss in average sum-rate.
- Latency time scale (t_c) , a design parameter: if larger, larger averaging period, higher latency (schedule when hitting a really high peak)

REMARKS ON MUDIV

- Signalling:
 - UEs: Track their link quality (common pilot)
 - BS: Access to quality measurements (delay-free feedback channel)
- Delay in the feedback channel (ass.: delay&error free)
 - · Mismatch actual channel-measured channel
 - FIX: ↓ scheduling slots ⇒ ↑ signalling overhead ⇒ selective MUDiv (f/b iff above threshold)
- · Fairness & delay:
 - Non-homogeneous user set in real-world networks (assumed so far)
 - Different statistics (Rayleigh, Rice,...) average SNRs (near-far).. RESOURCE ALLOCATION ??
 - FIX: Proportional Fair Scheduler (PFS)
- · Limited and slow fluctuations (ass: high & fast)
 - Limited: poor scattering/LOS Slow: low mobility environment
 - Result: low cell throughput (peaks) Delay requirements not met.
 - · FIX: Opportunistic beamforming.

OPPORTUNISTIC BEAMFORMING

- Slow fading hurts: If all users fade slow \Rightarrow like K=1 user \Rightarrow no MUDiv
- · Limited fluctuation hurts: lower peak rates

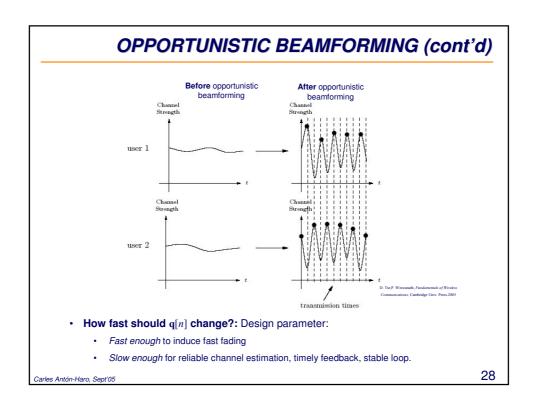


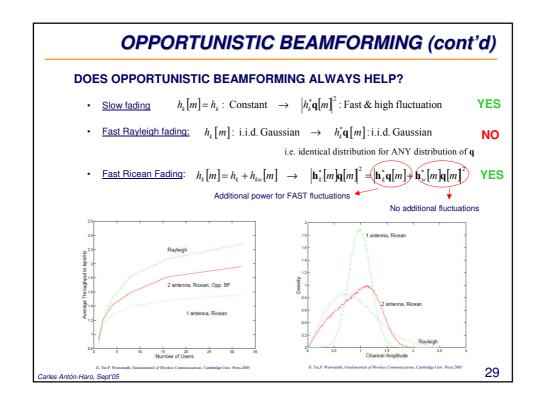
• TRICK (MISO): Induce fast and high fluctuations by transmit beamforming with a timevarying common set of random weights (e.g circularly symmetric Gaussian):

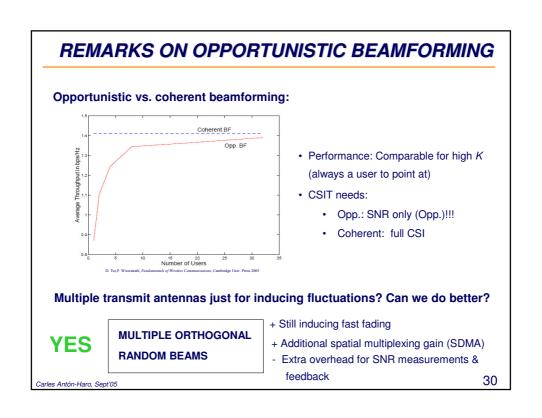


When are SNR peaks reached?: When beam "points" at user k

 $\mathbf{q}[m]/\!\!/\,\mathbf{h}_k^*[m]$ "OPPORTUNISTIC BEAMFORMING"







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 - ✓ Fairness issues: Proportional Fair Scheduling
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- Channel- and queue-aware scheduling
 - Motivation.
- □ Q&A

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CHANNEL- AND QUEUE-AWARE SCHEDULING

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ASSUMPTIONS REVISITED

- · Implicit assumptions so far...
 - Ass. 1: Infinite transmit buffer size:
 - Users can be delayed without bound (to maximize sum-rate).
 - Did not care much about packet arrival rates.
 - Ass.2 : Scheduled user(s) always have data to transmit
- BUT in realistic scenarios...
 - Finite buffer size:
 - When close to buffer overflow, user should be scheduled regardless of channel conditions.
 - If too many packets arrive, buffer bound to explode.
 - Traffic is bursty: no point in scheduling a user with empty buffer!
- CONCLUSION: Channel and queue (buffer) information must be jointly considered in the scheduling process (i.e. cross-layer)

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QUESTIONS?

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